

THE RISE AND FALL OF THE NATURAL INTEREST RATE

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The natural interest rate (r^*)

The real interest rate consistent with full employment & no nominal rigidities (*Woodford, 2003*)

- **A relevant concept for the conduct of monetary policy:**

It serves as optimal target

The central bank should set the nominal interest rate in order to close the real interest rate gap ($r - r^*$), thereby closing the output gap and stabilizing inflation



It gauges the stance of monetary policy

- contractionary if $r > r^*$
- expansionary if $r < r^*$

- **Being r^* not directly observable, it has to be inferred from the data**

Available estimates suggest that r^* stands at historically low (or possibly negative) levels

However, the conventional view is that estimates of r^* are very imprecise

Question 1: why so large uncertainty in r^* ?

We dig into the mechanics of the workhorse tool to estimate r^*

Holston, Laubach, & Williams (2017) model, hereafter **HLW**

2-equations model inspired on the New Keynesian framework

Phillips curve

*inflation depends on
output gap*

IS curve

*output gap depends on interest
rate gap ($r - r^*$)*

Large uncertainty arises in 2 cases: when either the IS or the Phillips curve is flat

**Data contain no info about the
unobserved states of the model**

The model fails to meet the observability
condition (Kalman, 1960)

**These cases are empirically relevant
(more the rule than the exception)**

Question 2: how to precisely measure r^* ?

Observation:
the HLW model treats the observed
real interest rate as exogenous



Hence the dynamic properties of interest rate
gap & output gap are unspecified

Nothing guarantees the stationarity of both gaps!



We consider an augmented HLW model to make both gaps stationary

The extra-equation is a **local level** specification for the observed real rate

$$\underbrace{r_t}_{\text{observed real rate}} = \underbrace{r_t^*}_{\text{natural rate } I(1)} + \underbrace{\tilde{r}_t}_{\text{interest rate gap } I(0)}$$

The model identifies r^*
even with flat IS & Phillips
curves

Interestingly, the univariate local level model can also identify r^*

Cons: it says nothing about drivers of r^*

since it exploits data on interest rate only

Pros: it always precisely estimate r^*

since it always meets observability, so it is robust
when data imply flat IS & Phillips curves

International evidence on r^*

We collect historical data (yearly frequency 1891-2016) for 17 advanced economies

Data likely to produce flat IS
and Phillips curves

(due to breaks & low frequency)



We use the data as
testing ground for
the local level model



We document

- a general decline of r^* since the start of XX century until the 1960's
- a subsequent **rise and fall**, peaking around the end of the 1980's

What has driven the rise & fall of the natural interest rate?

Estimate a Panel ECM
which exploits panel
variation on key
determinants of r^*
(productivity growth,
demographics, risk)

The evolving **demographic composition** can
explain this rise & fall

Since the 1960's

**rise of young baby
boomers → r^* rises**

Once baby boom ends

**young share falls due
to ageing → r^* falls**

Road map

1. Why is the uncertainty on r^* so large?

- Uncertainty in the HLW model
- Observability in the HLW model

2. How to precisely estimate r^* ?

- The augmented HLW & the local level model
- International evidence on r^*

3. Conclusions

The Holston, Laubach, & Williams (2017) model

Two key equations inspired by the New Keynesian framework

Phillips curve

$$\pi_t = \alpha_\pi \pi_{t-1} + (1 - \alpha_\pi) \pi_{t-2,4} + \kappa \tilde{y}_{t-1} + \varepsilon_t^\pi$$

inflation

output gap

$$\tilde{y}_t \equiv y_t - y_t^*$$

potential output

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_t^{y^*}$$

trend growth

$$g_t = g_{t-1} + \varepsilon_t^g$$

IS curve

$$\tilde{y}_t = \alpha_{y,1} \tilde{y}_{t-1} + \alpha_{y,2} \tilde{y}_{t-2} - \gamma (r_{t-1} - r_{t-1}^*) + \varepsilon_t^{\tilde{y}}$$

real interest rate

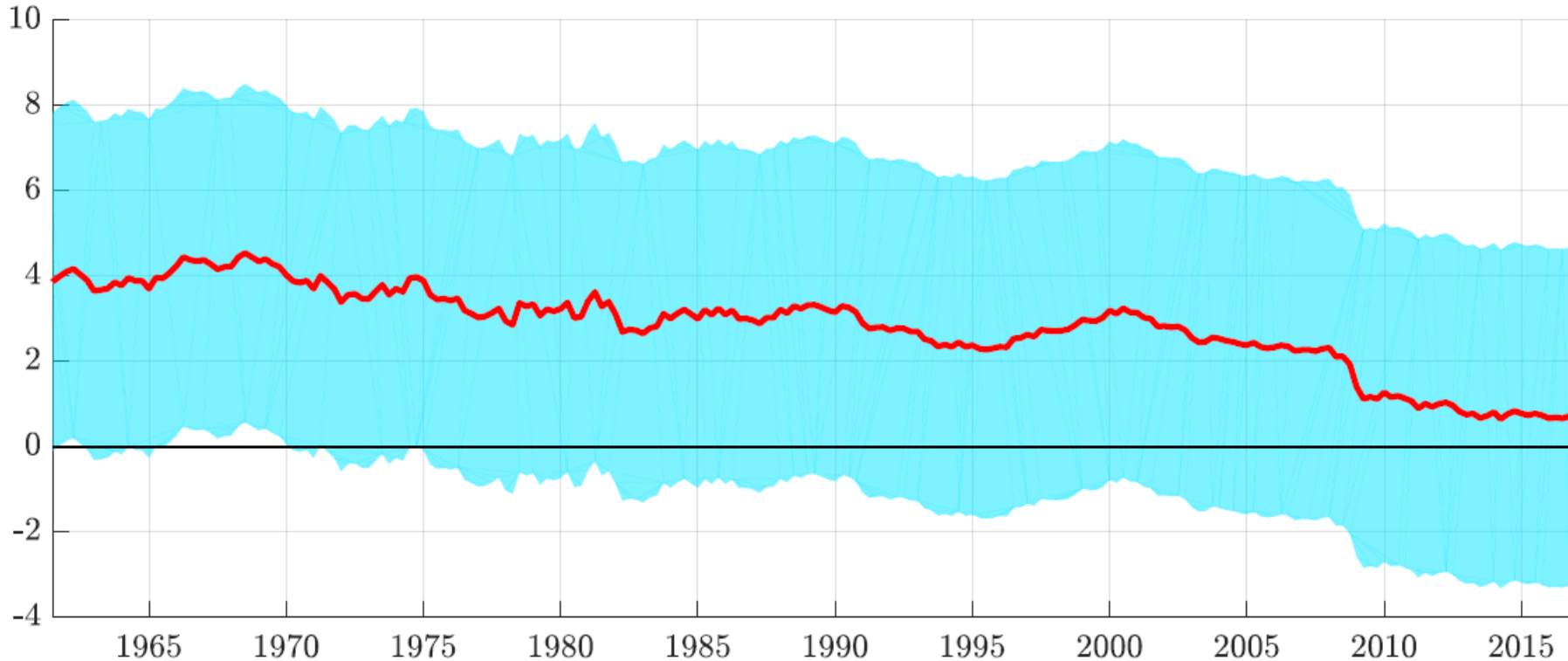
natural rate

$$r_t^* = 4g_t + z_t$$

unobserved factors
unrelated to growth

$$z_t = z_{t-1} + \varepsilon_t^z$$

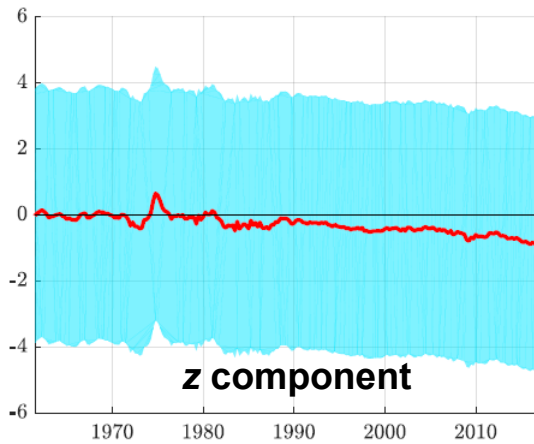
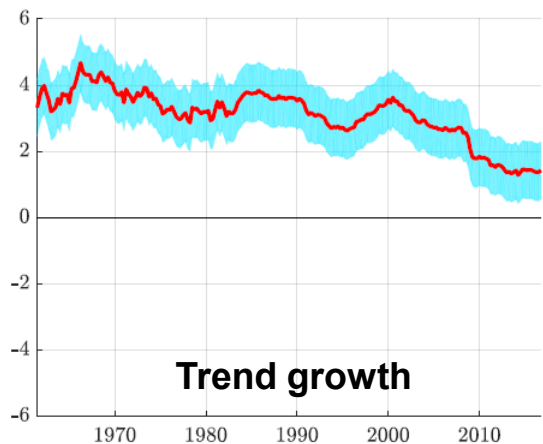
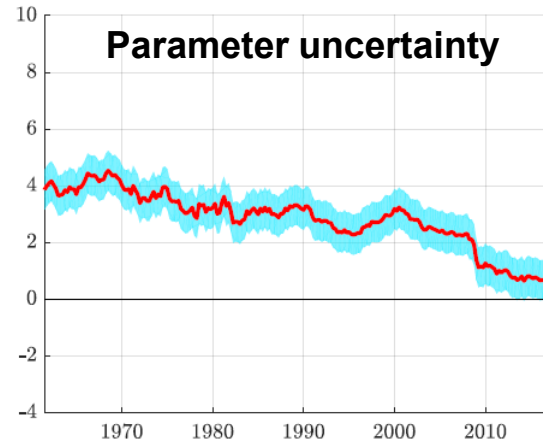
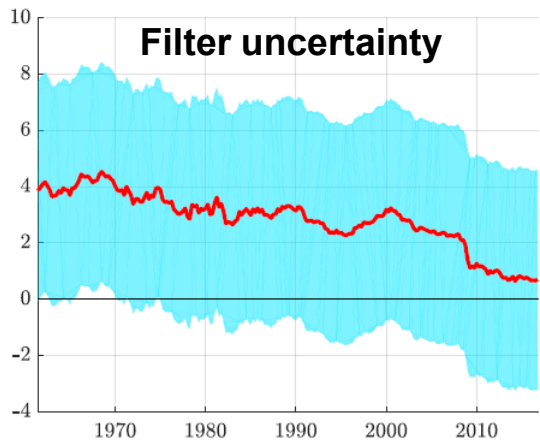
High uncertainty in estimated U.S. r^* by HLW model



Notes: 1961Q2:2016Q3, one-sided filter with 90% bands (both parameter and filter uncertainty)

Why so large uncertainty?

Large uncertainty is mostly due to filter uncertainty...



... and the large filter uncertainty stems from the z component

Observability in the HLW model

Measurement equation

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \underbrace{\begin{bmatrix} 1 - \alpha_y & 1 + 4\gamma & \gamma \\ -\kappa & 0 & 0 \end{bmatrix}}_{\mathbf{Z}} \begin{bmatrix} y_{t-1}^* \\ g_{t-1} \\ z_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} \alpha_y & 0 & 0 & -\gamma \\ \kappa & \alpha_\pi & 1 - \alpha_\pi & 0 \end{bmatrix}}_{\mathbf{D}} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ \pi_{t-2|4} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} \tilde{\varepsilon}_t^{y^*} + \varepsilon_t^{y^*} \\ \varepsilon_t^\pi \end{bmatrix}$$

Transition equation

$$\begin{bmatrix} y_t^* \\ g_t \\ z_t \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{T}} \begin{bmatrix} y_{t-1}^* \\ g_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{y^*} \\ \varepsilon_t^g \\ \varepsilon_t^z \end{bmatrix}$$

Which conditions allow for recovering the state vector from the data?



Observability

The rank of the observability matrix equals the number of unobserved states
(Harvey, 1989)

$$\text{Rank} \begin{bmatrix} \mathbf{Z} \\ \mathbf{ZT} \\ \mathbf{ZT}^2 \\ \vdots \\ \mathbf{ZT}^{s-1} \end{bmatrix} = s$$

Observability in the HLW model

Given that the HLW model features three unobserved states, the observability matrix reads

$$O = \begin{bmatrix} Z \\ ZT \\ ZT^2 \end{bmatrix} = \begin{bmatrix} 1 - \alpha_y & 1 + 4\gamma & \gamma \\ -\kappa & 0 & 0 \\ 1 - \alpha_y & 2 + 4\gamma - \alpha_y & \gamma \\ -\kappa & -\kappa & 0 \\ 1 - \alpha_y & 3 + 4\gamma - 2\alpha_y & \gamma \\ -\kappa & -2\kappa & 0 \end{bmatrix}$$

Flat IS curve $\gamma = 0$

$$O = \begin{bmatrix} 1 - \alpha_y & 1 & 0 \\ -\kappa & 0 & 0 \\ 1 - \alpha_y & 2 - \alpha_y & 0 \\ -\kappa & -\kappa & 0 \\ 1 - \alpha_y & 3 - 2\alpha_y & 0 \\ -\kappa & -2\kappa & 0 \end{bmatrix}$$

Cannot identify the z process

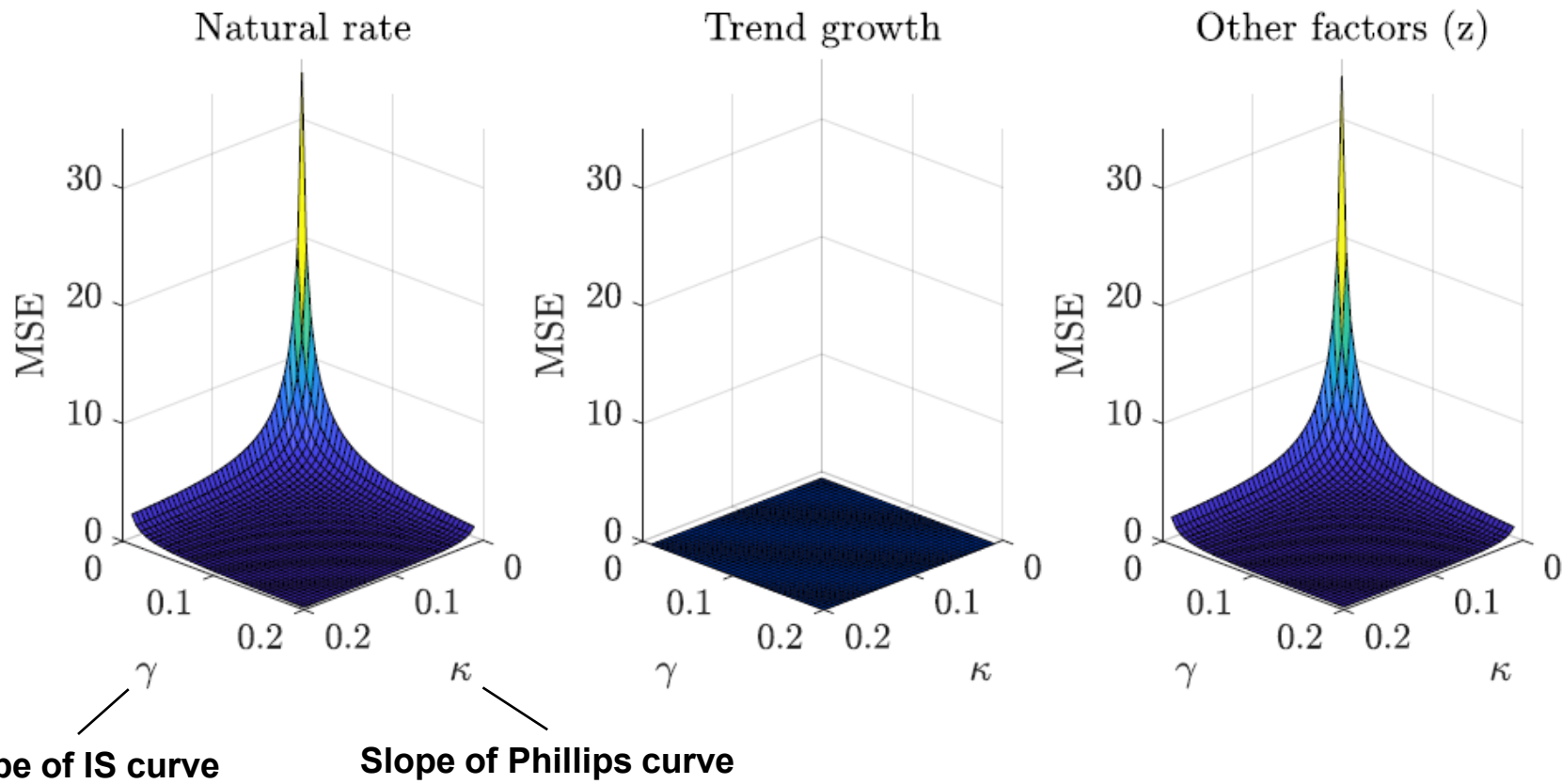
Flat Phillips curve $\kappa = 0$

$$O = \begin{bmatrix} 1 - \alpha_y & 1 + 4\gamma & \gamma \\ 0 & 0 & 0 \\ 1 - \alpha_y & 2 + 4\gamma - \alpha_y & \gamma \\ 0 & 0 & 0 \\ 1 - \alpha_y & 3 + 4\gamma - 2\alpha_y & \gamma \\ 0 & 0 & 0 \end{bmatrix}$$

Cannot separately identify z and y^*

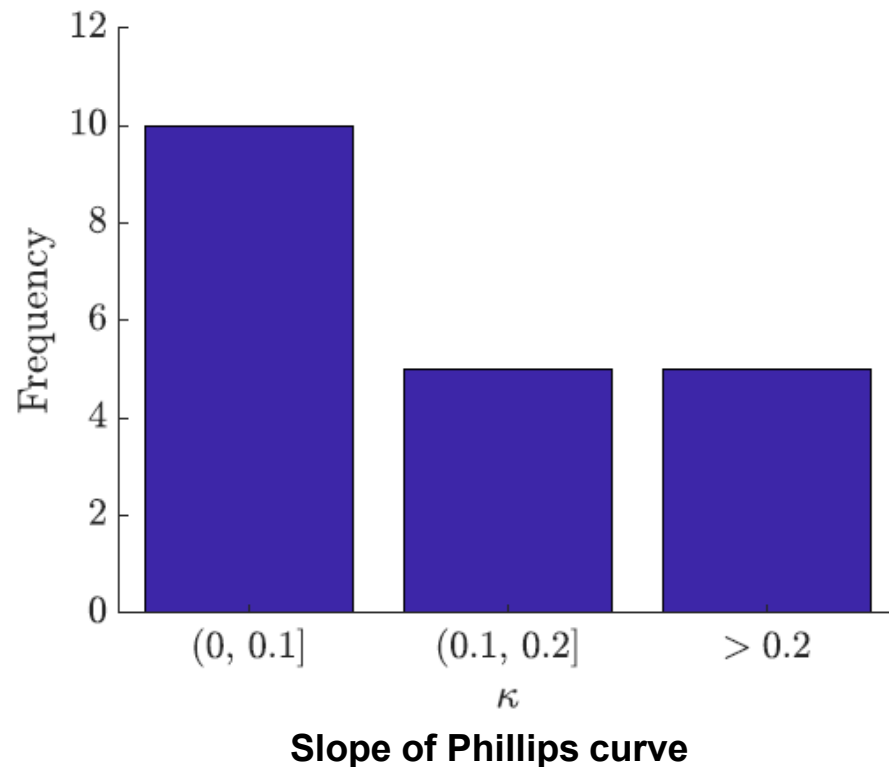
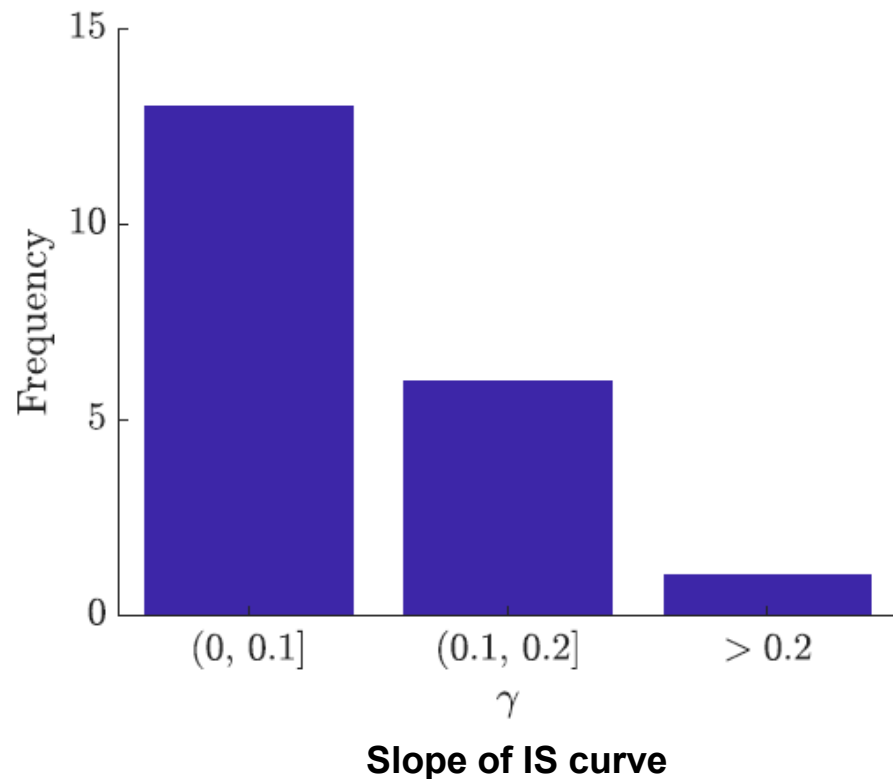
This matrix is rank deficient when the IS and/or the Phillips curves are flat

Filter uncertainty of HLW model & slopes of IS, Phillips curves



IS and Phillips curves are generally flat

Steepness of IS and Phillips curves: estimates in the literature



Road map

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- The augmented HLW & the local level model
- International evidence on r^*

3. Conclusions

The augmented HLW & the local level model

The HLW model treats the observed real interest rate as exogenous

$$\alpha_y(L)\tilde{y}_t = -\gamma(r_{t-1} - r_{t-1}^*) + \varepsilon_t^{\tilde{y}}$$

Hence the dynamic properties of both interest rate gap & output gap are unspecified

(gaps may be nonstationary!)

We consider an augmented HLW model to make both gaps stationary

The extra-equation is a **local level** specification for the observed real rate

$$\underbrace{r_t}_{\text{observed real rate}} = \underbrace{r_t^*}_{\text{natural rate } I(1)} + \underbrace{\tilde{r}_t}_{\text{interest rate gap } I(0)}$$

The model identifies r^* even with flat IS & Phillips curves

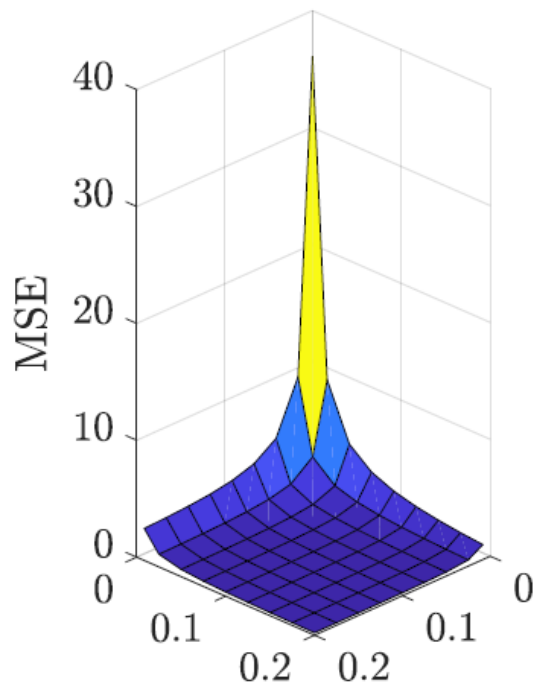
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Cons: it says nothing about drivers of r^* since it exploits data on interest rate only

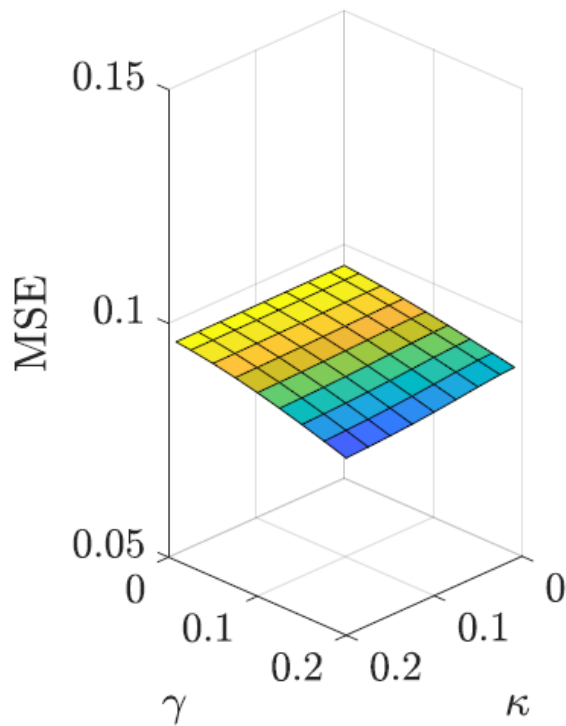
Pros: it always precisely estimate r^* since it always meets observability

Filter uncertainty of r^* across models

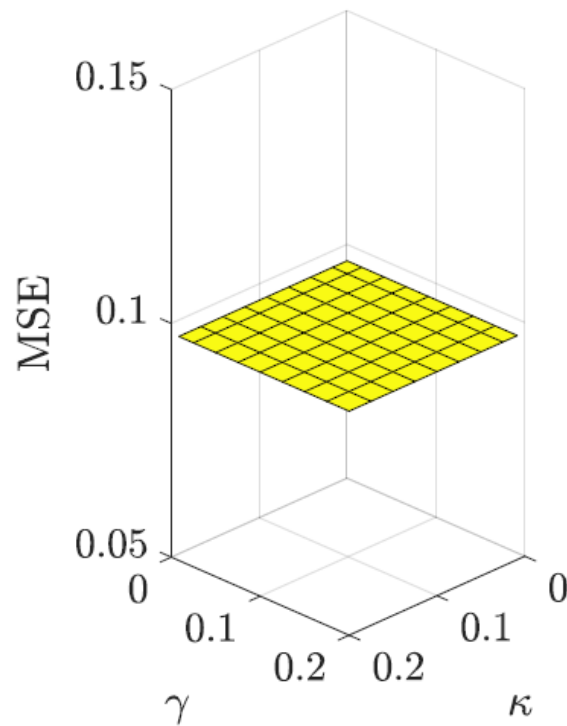
HLW model



Augmented HLW model



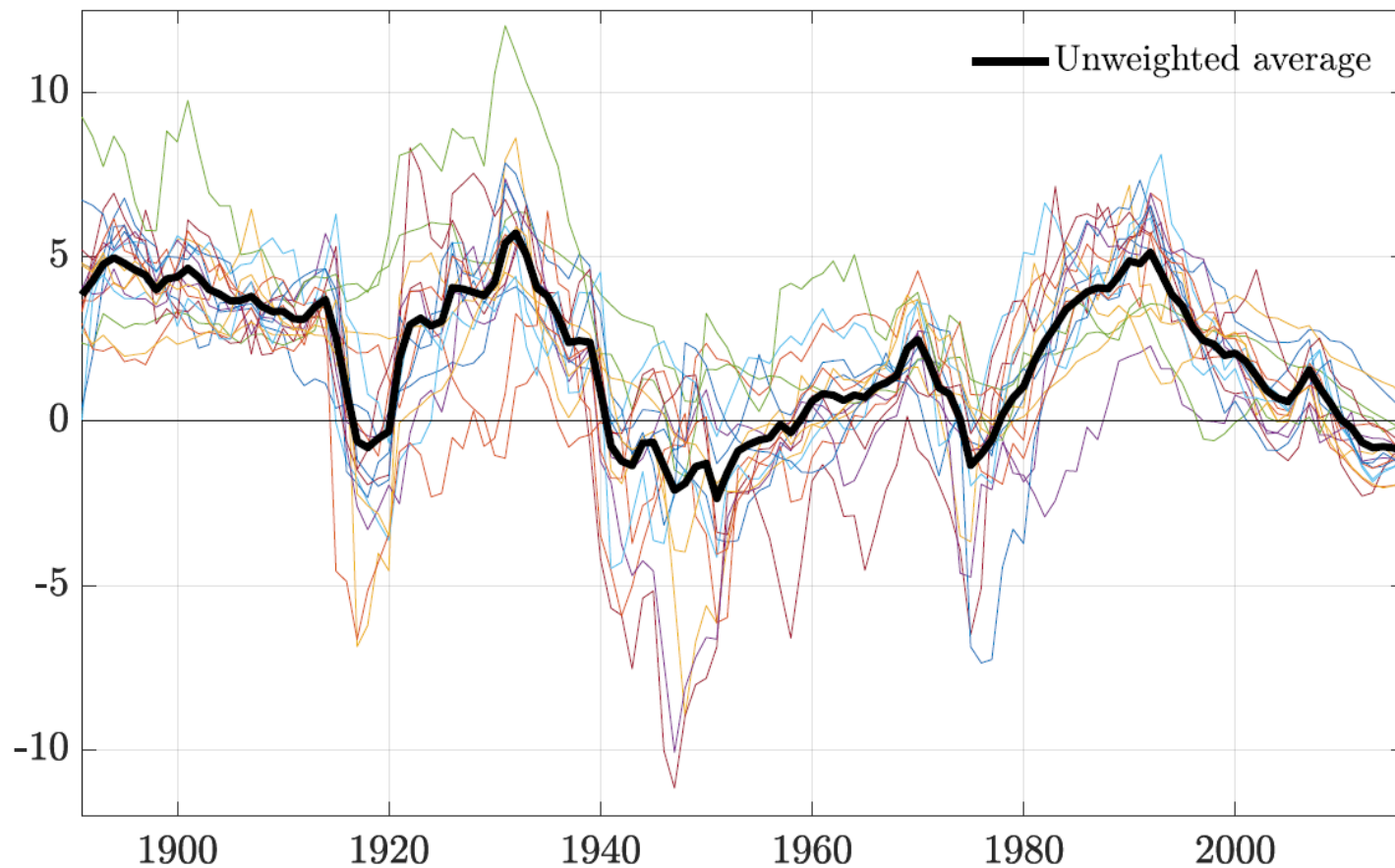
Local level model



γ
Slope of IS curve

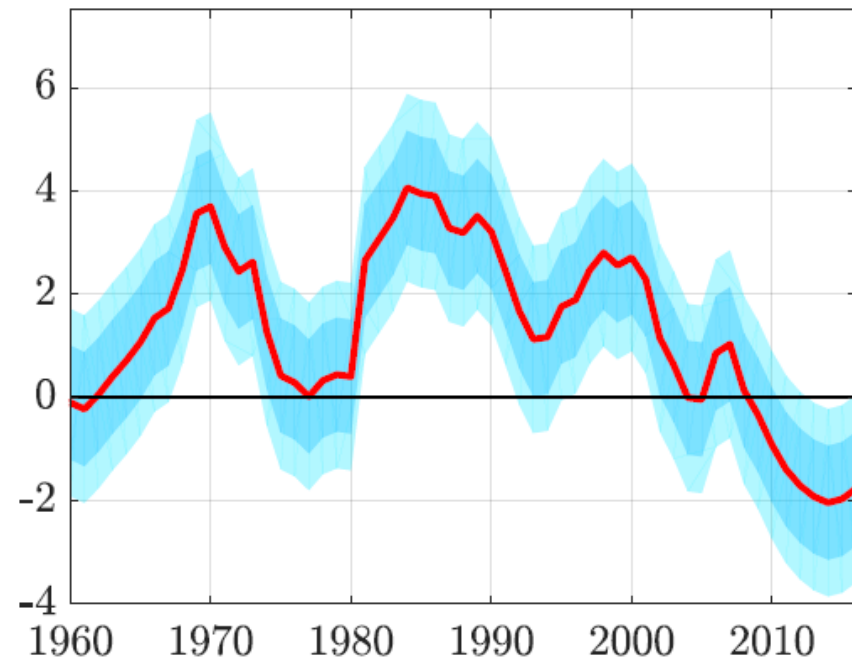
κ
Slope of Phillips curve

International evidence on estimated r^* by the local level model

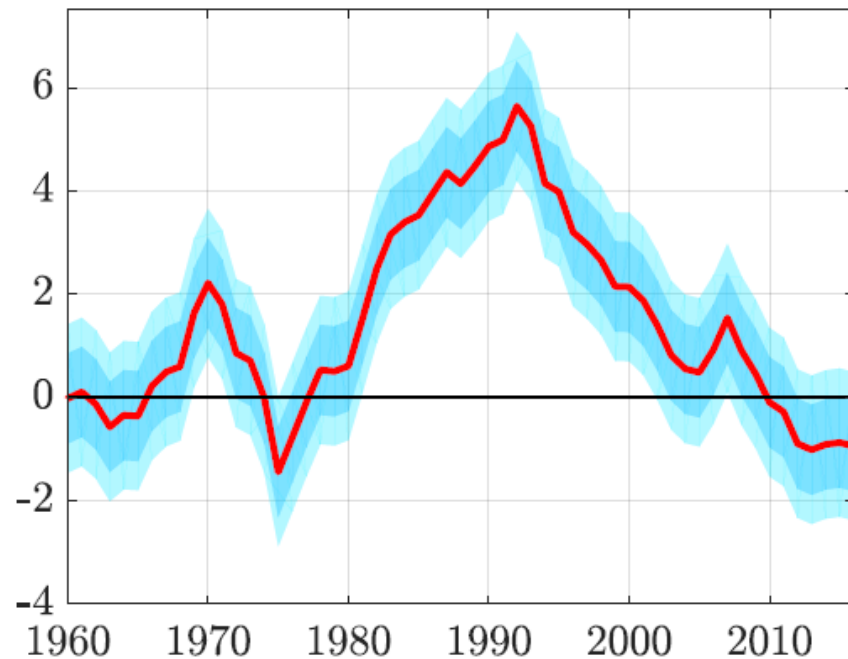


U.S. and euro area natural rates

United States

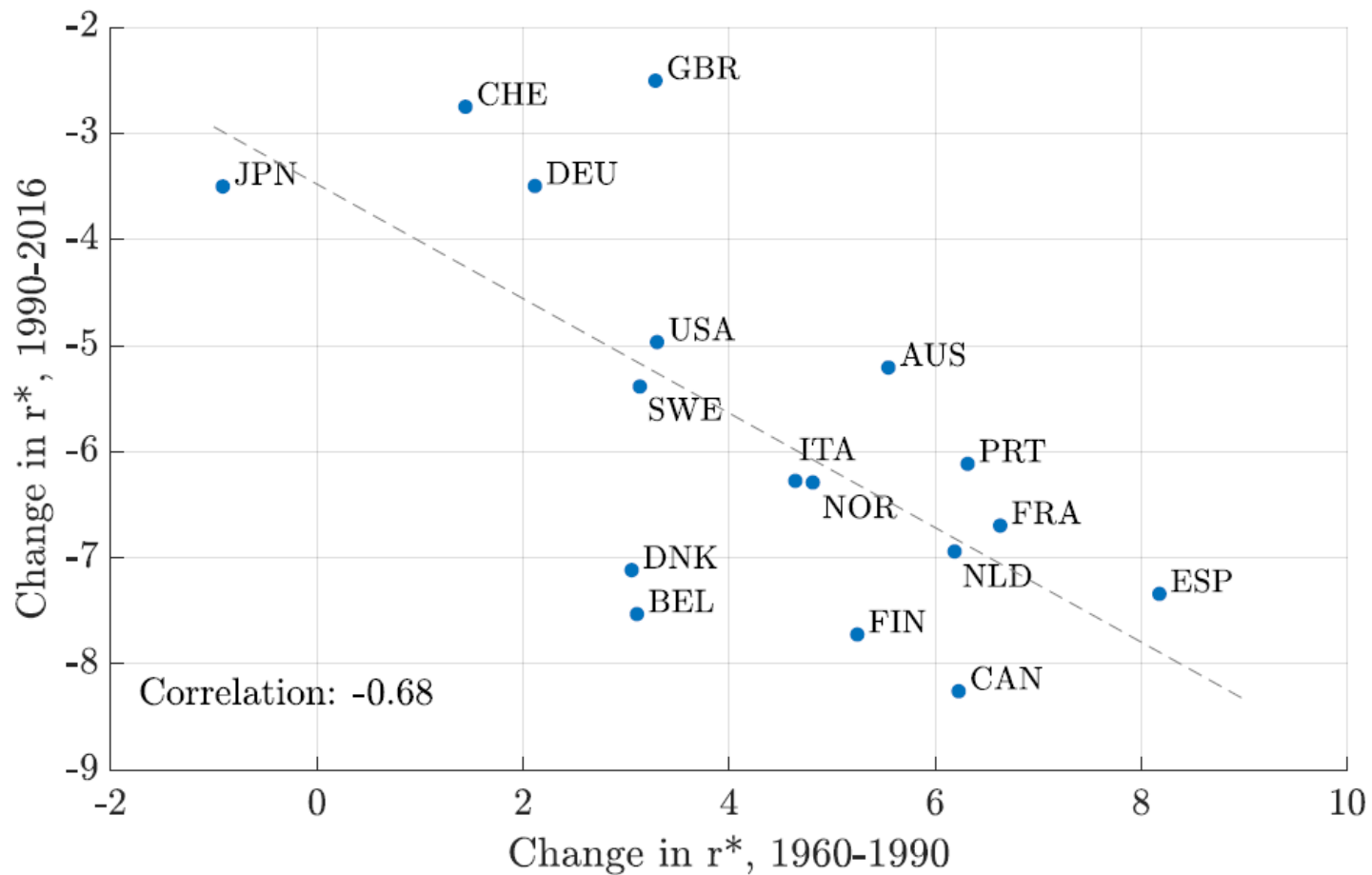


euro area



Notes: median estimates with 68% and 90% bands (both parameter and filter uncertainty)

What has driven the rise and fall in r^* ?



The r^* of the local level model is silent about its drivers



We consider an alternative but complementary approach by estimating a Panel ECM

$$\Delta r_{i,t} = \alpha [r_{i,t-1} - \beta' X_{i,t-1}] + \gamma' \Delta X_{i,t} + \varepsilon_{i,t}$$

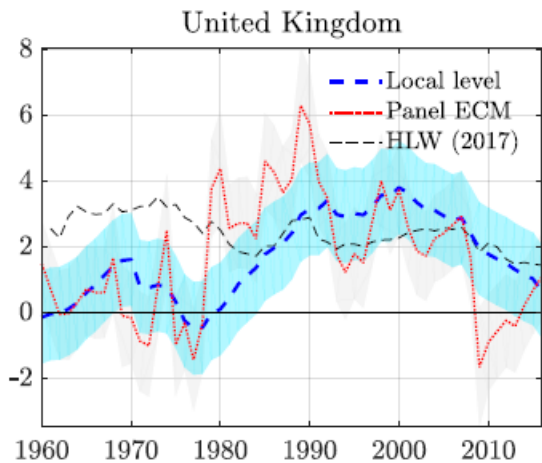
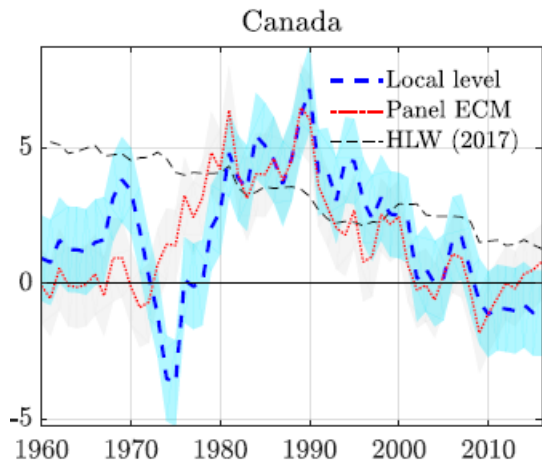
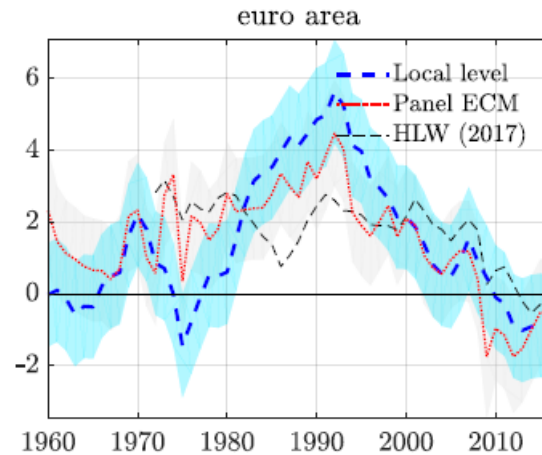
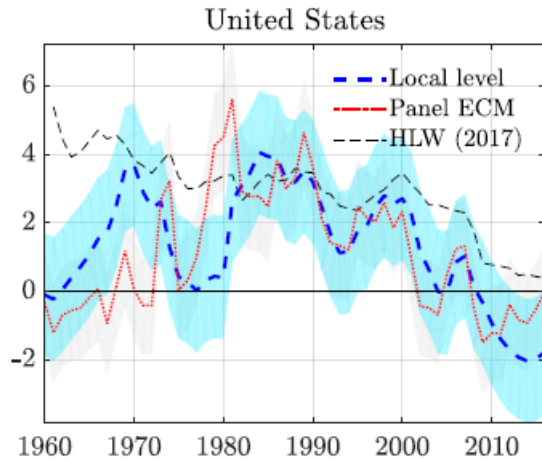
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real interest rate

(annual data, 1960-2016)

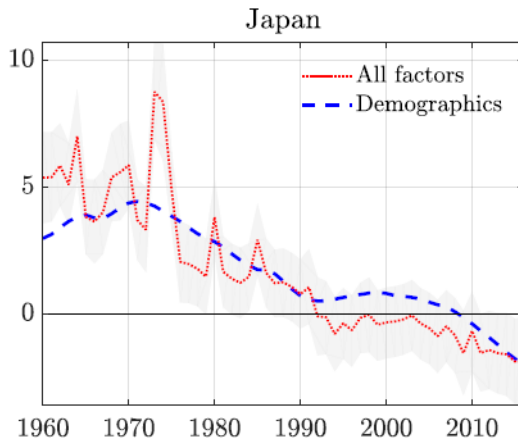
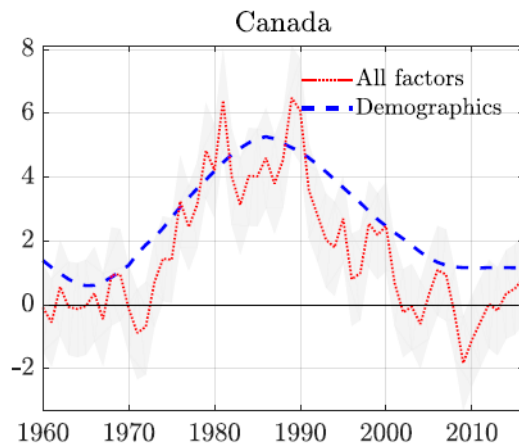
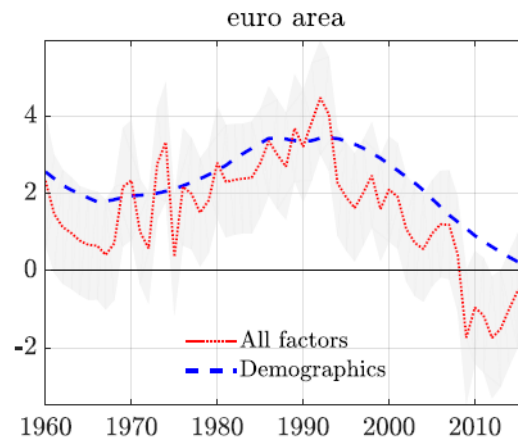
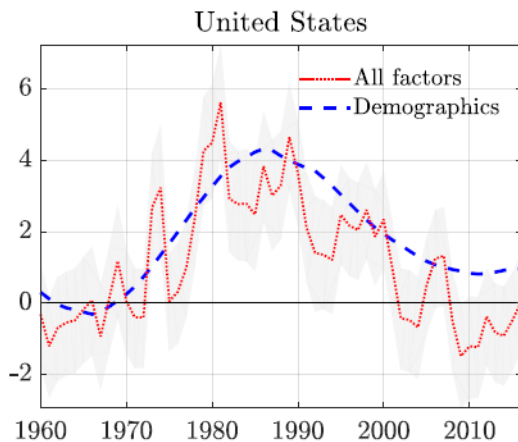
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indicators of potential determinants of r^*

- productivity (TFP) growth
- demographics (young share in population)
- risk (term spread)

Estimated r^* by Panel ECM



The role of demographics



Why is the uncertainty on r^* so large?

The precision of the HLW model dramatically drops with flat IS and/or Phillips curves

These cases appear to be more the rule than the exception

How to precisely estimate r^* ?

Augmented HLW model

which guarantees stationarity of rate & output gaps

Local level model

on the observed real interest rate

Using historical panel data we show a rise and fall of r^*

r^* rises since the 1960's and peaks
around the end of the 1980's

The evolving demographic composition
can explain part of this rise & fall

Thank you very much for your attention!